Efficiency Wage (and Slavery) Efficiency: in Theory and in Time

Alessandro SACCAL
Independent Researcher, Italy
saccal.alessandro@gmail.com

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Abstract:

The formal differentiation of (i) pain incentives from ordinary rewards, (ii) of effortful from careful production and (iii) of diligent from slothful workers under labor market imperfect competition ultimately suggests that the optimal menu of contracts associates inducements to production kinds following the preference triggered by slothful workers: effortful production with pain incentives and careful production with ordinary rewards. The efficiency of the efficiency wage as interpreted by the sociological theory is therefore discerned to arise under a particular production kind and so is that of slavery its dual (undoubtedly illicit). More broadly, the confusion of the two production kinds under market and state capitalism respectively contributes to the Phillips curve and price rigidity, in the misapplication of ordinary rewards to effortful production. State capitalism jurisprudentially eliminates the risks of dismissal and redundancy and thereby lastly causes effortful production to enter stagnation.

Keywords: care; diligence; efficiency; effort; pain; production; rewards; scourging; slavery; sloth; wages.

JEL Classification: D24; D86; E31; N20; O43; P51.

Introduction

Fenoaltea on Slavery

Fenoaltea (1999) rationalized slavery as an uncertain and transitory mechanism of exchange between backward and developed areas, trading slaves after wherewithal and prior to foodstuffs in return for manufactures, as bilateral price and real wage readjustments and transportation costs might allow. Fenoaltea (1984) had also argued pain incentives and ordinary rewards as optimal respective contracts for effortful and careful production. If slavery were to have ever started, he can be consequently synthesized to have held, unbeknownst to him or not, it would have ended through the bilateral real wage readjustment due to the exhaustion of the slave populations in the backward areas, at constant transportation costs, net of the accidental underpinnings[1] and short of abolition; foremost, it would have ended in spite of its contractual optimality relative to the effortful production for which it would have been perdurably employed, all else again equal.

Our View

The Arab slave trade's persistence unto contemporary abolitionism, however, contradicts Fenoaltea's (1999) Boserupian mechanism. Upon having gauged feudalism and the Atlantic slave trade too, as Fenoaltea (1999) himself had worked towards the formulation of his model, we (2021) thus objected reasoning that slavery is not Boserupian and that it would have temporarily ended wherever dechristianization had arisen, perduring in its absence and eventually restarting in those lands no longer Christian, to last indefinitely worldwide. Demand for manufactures from developed areas was not the drive behind the slave trades, we (2021) argued, but the supply of slaves itself, ever of appetite in view of fallen human nature and instrumental towards the goal of dechristianization; similarly, wherewithal and foodstuffs were not inter-temporal export substitutes for slaves on the backward area's part, to our (2021) mind, but intra or inter-temporal complements, at constant transportation costs.

In such a light the end of feudalism in Protestantized Europe features a robust explanation, as do (i) the continuation of the Arab slave trade, whose world was never Christian, to our postmodern age and (ii)

1 With regard to the Atlantic slave trade, for instance, he (1999) posited remittance technology development as the counterfactual drive behind its cessation. In other words, according to him (1999), had it not been abolished the Atlantic slave trade would have ended, on account of market forces, because of the development in telecommunications enabling emigrant remittances, which would have continued to finance foreign manufactures, accidental to his (1999) theory of bilateral real wage readjustments and transportation costs stability.

contemporary globalization. Similarly, the Atlantic slave trade is counterfactually held to have continued unbound and precisely on account of the reason we (2021) surmised are Fenoaltea's (1984) pain incentives as optimal contracts for effortful production deemed perdurable. From an alternative perspective, even if slavery is optimal for effortful production and even if that effortful production is to remain in place greater interests suggesting slavery's abandonment could be at stake, which we (2021) contended being those of dechristianization.

The substance of slavery's persistence, in sum, seems tied to the incentive mechanism extensively underlying slavery (*e.g.* dechristianization, Boserupian trade), whereas its accidents appear to concern slavery's contractual expedience in relation to production kind on account of the said mechanism.

Contributions

Since Fenoaltea's (1984) prescription is most proximate to the efficiency wage literature but yet singularly lacks a mathematical formalization we have hereby made it our task to strive supplying it. The efficiency wage literature to which it speaks, in case, is in the seminal acceptation of Robert Solow (1979) and therefrom Akerlof and Yellen (1990) (*i.e.* the sociological theory). His (1984) contract menu in fact specifies the sufficient (and perhaps necessary) conditions under which the efficiency wage is effectively efficient, thereby advising against all of its misapplications and attendant repercussions in abstract and historical time; the same holds for slavery its dual. We thus hereby intend to add noteworthy remarks relative to historical applications and theoretical deductions of the efficiency wage theory (and more) as declined above.

1. Inducement, Production and Worker Differentiation

1.1 Scourging and Wages

Let us posit a production function $f : \mathbb{R} \to \mathbb{R}$ with output $y \in \mathbb{R},$ first degree homogeneous and twice continuously differentiable, increasing and concave in labor input function $l : \mathbb{R}_+^2 \to \mathbb{R}_+$ and labor augmenting technology function $t : \mathbb{R}_+^2 \to \mathbb{R};$ let t be increasing and concave in scourging and wages[2] $s,\ w \in \mathbb{R}_+;$ let l be decreasing s increasing in w and concave in both: $f \circ tl : \mathbb{R}_+^2 \to \mathbb{R}$ in $y = f[t(s,\ w)l(s,\ w)]$ such that,

$$\forall \alpha \in \mathbb{R},\ f[\alpha t(s,\ w)l(s,\ w)] = \alpha f[t(s,\ w)l(s,\ w)],\ f[t(s,\ w)l(s,\ w)] \in C^2,\ f_{l,\ t} > 0,$$
$$(f_{l,\ t})_{l,\ t} < 0,\ t_{s,\ w} > 0,\ (t_{s,\ w})_{s,\ w} < 0,\ l_s < 0,\ l_w > 0 \text{ and } l_{ss,\ ww} < 0.$$

For clarity: f's first degree homogeneity captures constant returns to scale and its increase and concavity in l and t mimic diminishing marginal returns; the labor augmentation of t is known as Harrod neutrality; the increase and concavity of t in s and w model pain incentives and ordinary rewards, respectively, with a natural ceiling on elicited productivity; the respective decrease and increase of l in s and w and its concavity therein model a monopolistic labor supply setting an implicit markdown upon s and an implicit markup upon w in the provision of $l,$ deceleratively approached, relative to what marginal cost s and w at full employment would otherwise yield[3].

s is representative of pain incentives and therefore entails the threat of dismissal or, more broadly, of the risk of losing the means for the acquisition of subsistence; w is representative of ordinary rewards and therefore entail bonuses and *ad hoc* schemes of motivation, more broadly. It follows that pain incentives and ordinary rewards cannot perfectly substitute each other, even if production were not delegated, for effortful production requires different inducements from careful production, but more anon.

1.2 Costs and Profit

The firm is institutionally representative of all producers, that is, across all particular activities in time and space. Firm nominal profit is normally $\Pi = py - Wl(s,\ w) - F,$ where $p \in \mathbb{R}_{++}$ are prices, $W = pw$ nominal wages and $F \in \mathbb{R}_{++}$ is a fixed cost, but since both F and the variable cost, which prices $l,$ are hereby

2 $s,\ w \in \mathbb{R}$ could alternatively apply, although redundantly, for negative s and w respectively represent w and s on the positive line, that is, the worker respectively enjoys appropriation and suffers exploitation.

3 Production is not necessarily delegated, as the acting principal could preorder non-marginal cost pricing himself, even though it be improbable (being masochistic) in the event of an implicit s markup or w markdown (i.e. monopsonistic labor demand) whereby l increases in s and decreases in $w,$ all else equal. The virtue of diligence, opposed to the deadly sin of sloth, theoretically permits such a scenario, but it is ultimately clear that marginal cost s and w at full employment hinge upon the compromise between leisure and labor, on tastes and preferences, and that its alteration is a structural modification of societal norms and customs, fundamentally tied to the observance of no less than the natural law and thus impossibly relativistic.

functions of s and w they are directly formulated in real terms and so is firm profit therewith. Real variable cost[4] $c : \mathbb{R}_+^2 \to \mathbb{R}$ is non-increasing in s, increasing in w and concave in both: $\frac{c}{p} \equiv c(s,\ w)$ such that $c_s \leq 0,\ c_w > 0$ and $c_{ss,\ ww} < 0$. Real fixed cost $C : \mathbb{R}_+^2 \to \mathbb{R}_{++}$ is increasing and concave in s and w : $\frac{C}{p} \equiv C(s,\ w)$ such that $C_{s,\ w} > 0$ and $C_{ss,\ ww} < 0$. c and C are thus twice continuously differentiable and first degree homogeneous: $c,\ C \in C^2$ and, $\forall \alpha \in \mathbb{R},\ \alpha c(s,\ w) = c(\alpha s,\ \alpha w)$ and $\alpha C(s,\ w) = C(\alpha s,\ \alpha w)$. Real profit $\frac{\Pi}{p}$ is thus the difference between y, l valued at c and C : $\frac{\Pi}{p} = y - c(s,\ w)l(s,\ w) - C(s,\ w)$.

For clarity: the non-increase and concavity of c in s model bounded exploitation, that is, as s rises c can either remain constant or even fall, although meeting a floor[5]; the increase and concavity of c in w can be instantiated as taxation breaks, that is, as w rise c increases, but the firm could eventually enjoy taxation rebates; the increase and concavity of C in s and w model bounded supervision and negotiation costs, respectively, that is, hired labor features higher supervision costs than rented capital and rented capital features higher negotiation costs than hired labor[6] and as s or w rise to account for hired labor or rented capital C increases, though ever less, for supervision or negotiation is eventually formalized *(i.e.* internalized in cost).

1.3 Optimal Solutions

The firm therefrom determines the optimal solutions for s and w whenever the partial derivatives of $\frac{\Pi}{p}$ with respect to both be null, meaning changes are suboptimal. Since s and w are non-singular and the profit equation is concave the unconstrained maximization problem is both necessary and sufficient for optimal solutions.

- Non-linear programming problem.

$$\underset{\{s,\ w\}}{max} \frac{\Pi}{p} = f[t(s,\ w)l(s,\ w)] - c(s,\ w)l(s,\ w) - C(s,\ w).$$

- Marginal product of scourging.

$$\frac{\partial \Pi}{p \partial s} = 0 \longleftrightarrow f'(t_s l + t l_s) = (c_s l + c l_s) + C_s;$$

since $c \in \mathbb{R},\ l \geq 0,\ c_s \leq 0,\ l_s < 0$ and $C_s > 0$ we note that[7] $f'(t_s l + t l_s) > 0$ if and only if: (i) $-C_s < c_s l + c l_s < 0 \longleftrightarrow c_s l + c l_s \in (-C_s,\ 0)$; (ii) $-C_s < c_s l + c l_s = 0$; (iii) $-C_s < 0 < c_s l + c l_s \longleftrightarrow c_s l + c l_s \in (0,\ \infty)$. Positing $c,\ l > 0$, *ceteris paribus,* $f'(t_s l + t l_s) > 0$ if and only if $c_s l + c l_s \in (-C_s,\ 0)$.

- Marginal product of wages.

$$\frac{\partial \Pi}{p \partial w} = 0 \longleftrightarrow f'(t_w l + t l_w) = (c_w l + c l_w) + C_w;$$

since $c \in \mathbb{R},\ l \geq 0,\ c_w > 0,\ l_w$ and $C_w > 0$ we note that[8] $f'(t_w l + t l_w) > 0$ if and only if: (i) $-C_w < 0 < c_w l + c l_w \longleftrightarrow c_w l + c l_w \in (0,\ \infty)$; (ii) $-C_w < c_w l + c l_w = 0$; (iii)

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4 W is paid in currency and is therefrom a fraction of the positive nominal money supply M_S : $W \in M_S \subset \mathbb{R}_{++}$. Consequently, c models the fraction of the real money supply $m_s = M_s/p$ paying the worker: $c \in m_S$; the real variable cost would normally be w, but because it is also affected by s it is modelled more generally. Strictly speaking, in fact, the real variable cost should be c multiplied by l, that being price multiplied by quantity, *qua* negative revenue.

5 For instance, the worker is not paid his increased marginal product, but the same or ever less, unto starvation; eventually, he could be even stripped of his belongings, which have a limit.

6 Hired labor is consequently suitable over short periods, for supervision costs require little amortization, and rented capital over long periods, for negotiation costs are thereby better amortized.

7 Specifically, $(\overset{0,\ -}{c_s})(\overset{0,\ +}{l}) + (\overset{0,\ \pm}{c})(\overset{-}{l_s}) \longleftrightarrow (\overset{0,\ -}{c_s l}) + (\overset{0,\ \pm}{c l_s}) \longleftrightarrow (\overset{0,\ \pm}{c_s l + c l_s})$.

8 Specifically, $(\overset{+}{c_w})(\overset{0,\ +}{l}) + (\overset{0,\ \pm}{c})(\overset{+}{l_w}) \longleftrightarrow (\overset{0,\ +}{c_w l}) + (\overset{0,\ \pm}{c l_w}) \longleftrightarrow (\overset{0,\ \pm}{c_w l + c l_w})$.

$-C_w < c_w l + c l_w < 0 \longleftrightarrow c_w l + c l_w \in (-C_s,\ 0)$. Positing $c,\ l > 0$, *ceteris paribus,* $f'(t_w l + t l_w) > 0$ if and only if $c_w l + c l_w \in (0,\ \infty)$.

Assuming positive c and l to begin with (*i.e.* the real variable cost is borne and labor is thus present): the marginal product of s is positive if and only if the partial derivative of l priced in real terms with respect to s is negative, but greater than the negative partial derivative of C with respect to s (*i.e.* the decrease of priced l must exceed the negative increase of supervision costs); the marginal product of w is positive if and only if the partial derivative of l priced in real terms with respect to w is positive. Such is intuitive; for positive c and l : the decrease of l in s, suggesting lower y, is compensated by the decrease of c in s, outweighing the increase of C in s; the increase of c in w is compensated by the increase of l in w, producing higher y.

In a word, given positive c and l : if pain incentives rise then in order for y to increase priced l must decrease and more than offset the increase of supervision costs; if ordinary rewards rise then in order for y to increase priced l must increase.

1.4 Effortful and Careful Production

Let us differentiate effortful from careful production such that, all else equal, for effortful production ef it arises that t is increasing in s, non-increasing in w and concave in both and for careful production cf it arises that t is non-increasing in s, increasing in w and concave in both:

$$f[t(s,\ w)l(s,\ w)] = \begin{cases} ef[t(s,\ w)l(s,\ w)],\ t_s > 0,\ t_w \leq 0,\ t_{ss,\ ww} < 0 \\ cf[t(s,\ w)l(s,\ w)],\ t_s \leq 0,\ t_w > 0,\ t_{ss,\ ww} < 0 \end{cases},$$

ceteris paribus. ef and cf are such that pain incentives and ordinary rewards are respective apposite inducements therefor: $y_{ef} = ef[t(s,\ w)l(s,\ w)]$ and $y_{cf} = cf[t(s,\ w)l(s,\ w)]$ are best induced through s and w, respectively. The firm can be consequently divided into two institutional components, effortful and careful, and its real profits accordingly:

$$\frac{\Pi}{p} = \begin{cases} \frac{\Pi_{ef}}{p} = y_{ef} - c(s,\ w)l(s,\ w) - C(s,\ w) \\ \frac{\Pi_{cf}}{p} = y_{cf} - c(s,\ w)l(s,\ w) - C(s,\ w) \end{cases}.$$

1.5 Optimal Differentiated Solutions

All else equal, the differentiated unconstrained maximization problems are necessary and sufficient for optimal solutions.

- Non-linear programming problems.

$$\max_{\{s,\ w\}} \begin{cases} \frac{\Pi_{ef}}{p} = ef[t(s,\ w)l(s,\ w)] - c(s,\ w)l(s,\ w) - C(s,\ w) \\ \frac{\Pi_{cf}}{p} = cf[t(s,\ w)l(s,\ w)] - c(s,\ w)l(s,\ w) - C(s,\ w) \end{cases}.$$

- Marginal products of scourging.

$\frac{\partial \Pi_{ef}}{p \partial s} = 0 \longleftrightarrow ef'(t_s l + t l_s) = (c_s l + c l_s) + C_s$ and $\frac{\partial \Pi_{cf}}{p \partial s} = 0 \longleftrightarrow cf'(t_s l + t l_s) = (c_s l + c l_s) + C_s$; since $c_s \leq 0,\ l_s < 0$ and $C_s > 0$, positing $c,\ l > 0$, we note that:

$ef'(t_s l + t l_s) > 0$ if and only if $-C_s < c_s l + c l_s < 0 \longleftrightarrow c_s l + c l_s \in (-C_s,\ 0)$;

$cf'(t_s l + t l_s) \leq 0$ if and only if $c_s l + c l_s \leq -C_s < 0 \longleftrightarrow c_s l + c l_s \in (-\infty,\ -C_s]$.

- Marginal products of wages.

$\frac{\partial \Pi_{ef}}{p\partial w} = 0 \longleftrightarrow ef'(t_w l + tl_w) = (c_w l + cl_w) + C_w$ and $\frac{\partial \Pi_{cf}}{p\partial w} = 0 \longleftrightarrow cf'(t_w l + tl_w) = (c_w l + cl_w) + C_w;$ since $c_w > 0,\ l_w > 0$ and $C_w > 0,$ positing $c,\ l > 0,$ we note that:

$ef'(t_w l + tl_w) \not\leq 0,$ since $0 < c_w l + cl_w \leq -C_w < 0,$ thus, $ef'(t_w l + tl_w) > 0$ if and only if $-C_w < 0 < c_w l + cl_w \longleftrightarrow c_w l + cl_w \in (0,\ \infty);$

$cf'(t_w l + tl_w) > 0$ if and only if $-C_w < 0 < c_w l + cl_w \longleftrightarrow c_w l + cl_w \in (0,\ \infty).$

Positive c and l are again assumed to begin with. The marginal product of s for ef and the marginal product of w for cf are positive at the same necessary and sufficient conditions. The marginal product of s for cf is non-positive if and only if the partial derivative of l priced in real terms with respect to s is negative, but at most equal to the negative partial derivative of C with respect to s (*i.e.* the decrease of priced l is at most exceeded by the negative increase of supervision costs); the marginal product of w for ef cannot be non-positive since the partial derivative of l priced in real terms with respect to w is positive, but at most equal to the negative partial derivative of C with respect to $w,$ which is negative. Such is intuitive; for positive c and l : s and w respectively induce y_{ef} and y_{cf} as though production were not differentiated; for cf the decrease of l in $s,$ suggesting lower $y,$ is compensated by the decrease of c in $s,$ though insufficiently outweighing the increase of C in $s;$ for ef the increase of c in w is compensated by the increase of l in $w,$ producing higher $y,$ all the same.

In a word, given positive c and l : if pain incentives rise then in order for y to increase priced l must decrease and more than offset the increase of supervision costs, though only for $ef,$ as pain sabotages care altogether; if ordinary rewards rise then in order for y to increase priced l must increase, even for $ef,$ as rewards facilitate effort withal.

1.6 Diligence, Sloth and Type Marginal Products

Let us differentiate diligent from slothful workers such that for the probability space $(\Omega,\ \Theta,\ \rho)$: Ω is the sample space; $\Theta = \{\theta_D,\ \theta_S\} \subset \mathcal{P}(\Omega)$ is the σ-algebra; $\rho : \Theta \to [0,\ 1]$ is the probability measure originating the probability mass function $\rho_i = \rho(\theta_i),\ \forall i = D,\ S,$ with $\rho_i \geq 0$ and $\sum_{i=D}^{S} \rho_i = 1.$

Diligent workers operate as though production were not differentiated such that pain incentives perfectly substitute ordinary rewards[9]. For positive c and $l,$ the positive marginal product of s of the diligent worker thus equals his positive marginal product of $w,$ notwithstanding production kind:

$ef'(t_s l + tl_s) = ef'(t_w l + tl_w) > 0$ if and only if $c_s l + cl_s \in (-C_s,\ 0)$ and $c_w l + cl_w \in (0,\ \infty);$

$cf'(t_s l + tl_s) = cf'(t_w l + tl_w) > 0$ if and only if $c_s l + cl_s \in (-C_s,\ 0)$[10] and $c_w l + cl_w \in (0,\ \infty).$

Slothful workers operate according to production differentiation. Under ef it is the case that t is increasing in $s,$ but non-increasing in $w,$ consequently, for positive c and $l,$ the marginal product of s of the slothful worker exceeds his marginal product of $w,$ which is positive; under cf it is the case that t is increasing in $w,$ but non-increasing in $s,$ consequently, for positive c and $l,$ the marginal product of s of the slothful worker is non-positive and exceeded by his marginal product of $w,$ also positive:

$ef'(t_s l + tl_s) > ef'(t_w l + tl_w) > 0$ if and only if $c_s l + cl_s \in (-C_s,\ 0)$ and $c_w l + cl_w \in (0,\ \infty);$

$cf'(t_s l + tl_s) \leq 0 < cf'(t_w l + tl_w)$ if and only if $c_s l + cl_s \in (-\infty,\ -C_s]$ and $c_w l + cl_w \in (0,\ \infty).$

1.7 Optimal Contracts

All else equal, the adverse selection *(i.e.* hidden information) relative to the worker type produces a necessary and sufficient unconstrained maximization problem for optimal solutions and contracts.

- Non-linear programming problem.

9 Such workers are pleonastically diligent owing to the virtue of diligence; in fact, they could be said to be diligently resigned. More broadly, they are so because of that which Catholic doctrine terms the gifts of fortitude and fear of God, respectively produced by enlightened human will and the Spirit of God.

10 Specifically, $cf'(t_s l + tl_s) > 0$ if and only if $-C_s < c_s l + cl_s < 0 \longleftrightarrow c_s l + cl_s \in (-C_s,\ 0),$ *ceteris paribus.*

$$\max_{\{s_D,\ S,\ w_D,\ S\}} \frac{\Pi}{p} = \rho_D f[t(s_D,\ w_D)l(s_D,\ w_D)] + \rho_S f[t(s_S,\ w_S)l(s_S,\ w_S)]$$
$$-\rho_D c(s_D,\ w_D)l(s_D,\ w_D) - \rho_S c(s_S,\ w_S)l(s_S,\ w_S) - \rho_D C(s_D,\ w_D) - \rho_S C(s_S,\ w_S).$$

- Marginal products of scourging.

$$\frac{\partial \Pi}{p\partial s_{D,\ S}} = 0 \longleftrightarrow \rho_{D,\ S} f'(t_{s_{D,\ S}} l + t l_{s_{D,\ S}}) = \rho_{D,\ S}(c_{s_{D,\ S}} l + c l_{s_{D,\ S}}) + \rho_{D,\ S} C_{s_{D,\ S}}$$ and
$\rho_{D,\ S} \in (0,\ 1]$, *ceteris paribus* $\longrightarrow f'(t_{s_{D,\ S}} l + t l_{s_{D,\ S}}) = (c_{s_{D,\ S}} l + c l_{s_{D,\ S}}) + C_{s_{D,\ S}}$.

- Marginal products of wages.

$$\frac{\partial \Pi}{p\partial w_{D,\ S}} = 0 \longleftrightarrow \rho_{D,\ S} f'(t_{w_{D,\ S}} l + t l_{w_{D,\ S}}) = \rho_{D,\ S}(c_{w_{D,\ S}} l + c l_{w_{D,\ S}}) + \rho_{D,\ S} C_{w_{D,\ S}}$$ and
$\rho_{D,\ S} \in (0,\ 1]$, *ceteris paribus* $\longrightarrow f'(t_{w_{D,\ S}} l + t l_{w_{D,\ S}}) = (c_{w_{D,\ S}} l + c l_{w_{D,\ S}}) + C_{w_{D,\ S}}$.

The (expected) marginal product of s of the diligent worker equals his (expected) marginal product of w if and only if the sum of his (expected) partial derivatives of priced l and C with respect to s equal the sum of those with respect w :

$$f'(t_{s_D} l + t l_{s_D}) = (c_{s_D} l + c l_{s_D}) + C_{s_D} = (c_{w_D} l + c l_{w_D}) + C_{w_D} = f'(t_{w_D} l + t l_{w_D}),$$

for $\rho_D \in (0,\ 1]$. For diligent workers there thus emerges indifference between pain incentives and ordinary rewards, between the hire of labor and the rent of capital, to the end of inducing production, be it effortful or careful.

The (expected) marginal product of s of the slothful worker is greater or smaller than his (expected) marginal product of w if and only if the sum of his (expected) partial derivatives of priced l and C with respect to s is respectively greater or smaller than the sum of those with respect to w : $f'(t_{s_S} l + t l_{s_S}) = (c_{s_S} l + c l_{s_S}) + C_{s_S} \gtrless (c_{w_S} l + c l_{w_S}) + C_{w_S} = f'(t_{w_S} l + t l_{w_S})$, for $\rho_S \in (0,\ 1]$. Specifically,

$$f = \begin{cases} ef \longrightarrow f'(t_{s_S} l + t l_{s_S}) > f'(t_{w_S} l + t l_{w_S}) \\ cf \longrightarrow f'(t_{s_S} l + t l_{s_S}) < f'(t_{w_S} l + t l_{w_S}) \end{cases}.$$

For slothful workers there thus emerges preference between pain incentives and ordinary rewards, between the hire of labor and the rent of capital, to the end of inducing production: if production is effortful then preference yields to pain incentives and the hire of labor; if production is careful then preference yields to ordinary rewards and the rent of capital.

The indifference between inducements for diligent workers allows for the association of s and w with y_{ef} and y_{cf}, respectively, as required by the optimal inducements of slothful workers; the optimal menu of contracts is consequently $(y_{ef},\ s)$ and $(y_{cf},\ w)$: if production is effortful then labor is hired and workers face pain incentives; if production is careful then capital is rented and workers face ordinary rewards. Deductively, in the acceptation of the sociological theory, cf is a sufficient (and perhaps necessary) condition for the effective efficiency of the efficiency wage and ef is a sufficient condition for the efficiency of slavery its dual.

2. Historical and Theoretical Remarks

2.1 Labor Market Perfect Competition

The introduction of perfect competition on the part of labor demand and supply simplifies $f = ef,\ cf$ such that $y = f[t(s,\ w)l]$ and $l \in \mathbb{R}_{++}$, all else equal. The absence of monopolistic labor supply hereby allows for marginal cost pricing such that l cannot decrease if s rises or w fall (*i.e.* no implicit s markdown or w markup) and l can no longer be therefrom reduced to no employment, full employment being indeed enjoyed. Under ef and cf the price of l is ever W on account of perfect competition in the labor market: in real terms, workers are assumed to bargain the fair pay of their marginal products in exchange for marginal cost s and w, themselves driven by the absence of monopolistic labor supply[11] and monopsonistic labor demand. The specific difference in relation to labor market imperfect competition is that the real variable cost is no function of s amounting to w

11 It could be argued that monopolistic labor supply ought to eliminate slavery altogether and that even under labor market perfect competition (whereby it would otherwise be present) it would hardly subsist, for neither employer would consider it; employers are yet hereby contended to desiderate it because of fallen human nature, once again. The concomitance of s and w is therefore not abstruse; moreover, it must be borne in mind that s are pain incentives touching not only the outright scourge, doubtless illicit, but the broader risks of dismissal and redundancy as well.

thereby[12]. Real profits are thus $\frac{\Pi_f}{p} = f[t(s,\ w)l] - wl - C(s,\ w),$ all else equal, and the first order conditions are $\frac{\partial \Pi_f}{\partial s} = 0 \longleftrightarrow lf_t t_s = C_s$ and $\frac{\partial \Pi_f}{\partial w} = 0 \longleftrightarrow lf_t t_w = l + C_w;$ indeed:

$ef_t t_s > 0 \longleftrightarrow \frac{C_s}{l} > 0 \longrightarrow C_s \in (0,\ \infty);$

$ef_t t_w \not> 0,$ since $ef_t > 0$ and $t_w \leq 0,$ *ceteris paribus,* and $ef_t t_w \leq 0 \longleftrightarrow 1 + \frac{C_w}{l} \leq 0 \longrightarrow C_w \leq -l,$ but $C_w,\ l > 0 \longrightarrow 0 < C_w \leq -l < 0,$ so $y \neq ef[t(s,\ w)l],$ *ceteris paribus,* but $y = ef[t(s)l];$

$cf_t t_s \not> 0,$ since $cf_t > 0$ and $t_s \leq 0,$ *ceteris paribus,* and $cf_t t_s \leq 0 \longleftrightarrow \frac{C_s}{l} \leq 0 \longrightarrow C_s \leq 0,$ but $C_s > 0,$ so $y \neq cf[t(s,\ w)l],$ *ceteris paribus,* but $y = cf[t(w)l];$

$cf_t t_w > 0 \longleftrightarrow 1 + \frac{C_w}{l} > 0 \longrightarrow C_w > 0 > -l \longrightarrow C_w \in (0,\ \infty).$

The marginal products of s and w for ef and $cf,$ respectively, are positive if and only if the partial derivatives of C with respect to s and w are positive, that is, supervision costs and negotiation costs respectively rise as labor is hired and capital is rented, as expected.

The marginal product of w and s for ef and $cf,$ respectively, cannot be positive since the partial derivative of ef and cf with respect to t is suitably positive and the partial derivative of t with respect to w and s is suitably non-positive. The marginal product of w and s for ef and $cf,$ respectively, can neither be non-positive, however, for: (i) the partial derivative C with respect to w is no more than negative $l,$ but since l and the partial derivative of C with respect to w are positive either the latter or the former is negative, contradicting either the assumption whereby negotiation costs increase in w or that whereby l may not be reduced to no employment; (ii) the partial derivative of C with respect to s is non-positive, but since it is positive it contradicts the assumption whereby supervision costs increase in s.

Consequently, for ef and cf it respectively arises that t is only increasing in s and $w,$ all else equal. In the presence of perfect competition in the labor market, in a word, the worker type adverse selection poses no problem and the unconstrained maximization problem for optimal contracts does not arise, being merely a differentiated one, for the inducements of s and w which monopolistic labor supply prescribes as optimal are naturally associated to ef and $cf,$ respectively, that is, in exclusivity: $\frac{\Pi_{ef}}{p} = ef[t(s)l] - wl - C(s)$ and $\frac{\Pi_{cf}}{p} = cf[t(w)l] - wl - C(w)$ only give rise to $(y_{ef},\ s)$ and $(y_{cf},\ w),$ respectively.

2.2 Phillips Curve, Price Rigidity and Steady State

In what follows, unless otherwise outlined, the steady state condition shall be synonymous with that of full employment, unlike in the Shapiro and Stiglitz (1984) acceptation of the efficiency wage by which offered w exceed full employment *(i.e.* market) ones in order for workers not to shirk, on the assumption that shirkers may be identified with a positive probability and therefrom dismissed, being at once unable to be re-employed at full employment $w,$ indicative of shirking. As a consequence, offered w are arguably identified with steady state ones, thereby diverging from full employment ones.

That clarified, we have seen that under ef and $cf,$ all else equal, t only increases in s and $w,$ respectively. Now, the confusion of the two production kinds explains the misapplication of ordinary rewards to $ef,$ the contract menu $(y_{ef},\ w),$ that is to say, which after a negative demand shock gives rise to (i) downwards stickiness and (ii) rigidity in prices under market and state capitalism, respectively. The contract menu $(y_{ef},\ w)$ and the annexed confusion are respectively accidental and substantial under market and state capitalism.

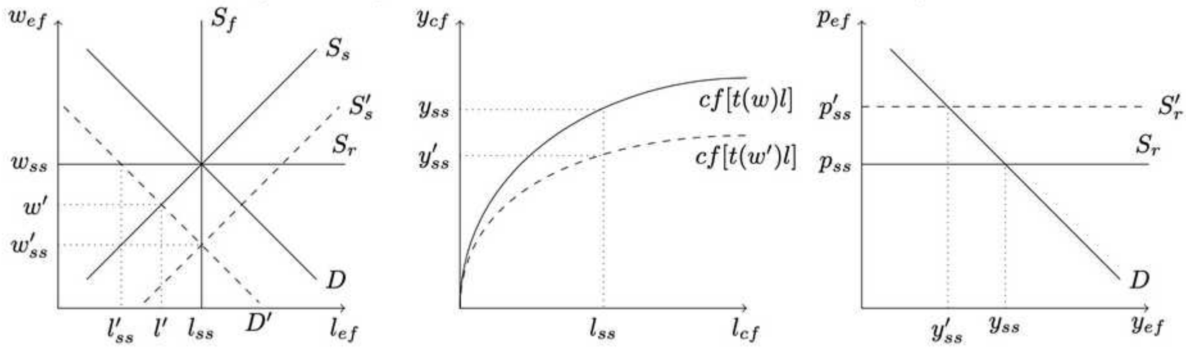
Under market capitalism a negative demand shock *(i.e.* lower quantity supplied) to y_{ef} decreases the demand for $l,$ which causes the marginal product of l to decrease in turn (*i.e.* lower opportunity cost of production) and the excess supply of l to enter redundancy; the supply of l at such prospected lower w ultimately increases

12 Under labor market imperfect competition employers certainly remunerate workers in real terms by means of w but their real variable cost is potentially decreased by s, as specifically outlined above; thence the pay being ultimately unfair.

(*i.e.* higher quantity demanded), allowing the workforce to return to full employment, at even lower w (*i.e.* lower opportunity cost of consumption). If the supply of l immediately accommodated the lower w then the aggregate supply function and the Phillips curve its dual would be vertical, not respectively increasing and decreasing, that is, there would be no lag in readjustment, which lag in readjustment (*i.e.* dismissal and eventual re-employment) is hereby posited to be precisely caused by the unwillingness to sufficiently decrease w on the part of employers in view of their misapplication of ordinary rewards to ef unbeknownst to them[13].

The firm's confusion of ef with cf specifically causes it to fear that said sufficient decrease in w may give rise to such a fall in y_{ef} as that brought about under cf, which lowers full employment y_{cf} by means of t (*i.e.* the steady state is negatively altered), whereas y_{ef} would actually return to full employment. Such a point was materially made by Fenoaltea (1984) and only in part even by Solow (1979), for the latter deemed y increasing in w altogether, absent differentiating y_{ef} from y_{cf} and thereby explaining why employers might choose not to decrease w as contended by the former, but rather dismiss and eventually re-employ at even lower, full employment w, that is to say: Solow (1979) surmised $y = f[t(w)l]$ and $f_t,\ t_w > 0,$ *ceteris paribus.* Downwards price stickiness can thus be hereby stated to be driven by employers, as opposed to the commonplace of workers, but if it were not for such a contended mechanism the unwillingness in question would be rather unlikely, nowadays especially.

Figure 1. Negative demand shock and institutional effects on y



Note. The first graph depicts the contract menu (y_{ef}, w) under market and state capitalism, respectively being accidental and substantial. Under market capitalism a negative demand shock to y_{ef} decreases l demand and w therewith (*i.e.* D' and w'), but employers confuse y_{ef} with y_{cf} and thereby refuse to sufficiently decrease w in order for l_{ss} to be preserved, fearing steady state y_{ef} may fall as well as if it were y_{cf}, which the second graph depicts (*i.e.* y'_{ss} due to w', from steady state w). The l supply is therefore not the flexible S_f, but the sticky S_s, the Phillips curve's inverse, broadly speaking. The excess l supply is thus dismissed (*i.e.* l') and eventually re-employed at lower w'_{ss}, which both workers and employers have by then accepted, returning to l_{ss} through a rise in l supply (*i.e.* S'_s). Under state capitalism the negative demand shock decreases l demand, but since w are institutionally rigid (i.e price rigidity, l supply S_r) l_{ss} and steady state y_{ef} permanently decrease (*i.e.* w_{ss} at l'_{ss}); the workforce however remains fully employed by statute, the excess supply whereof stands idle. Moreover, state capitalism's substantial contract menu (y_{ef}, w) causes a fall in s, no longer suffering the risks of dismissal and redundancy, which negatively affects y_{ef} through t, thereby being permanently reduced together with supply (*i.e.* y_{ss} and S_r fall), p_{ss} instead increasing, as depicted by the third graph (*i.e.* p'_{ss} at y'_{ss} through S'_r).

Under state capitalism nothing changes except for the fact that the excess supply of l does not enter redundancy by statute, that is, ordinary rewards are institutionally misapplied, thereby causing full employment y_{ef} to stagnate, until unemployment be allowed or price flexibility be reached even on the part of workers (although unnaturally, societal norms and customs being again at stake). To be sure, the supply of l at the new steady state permanently stagnates relative to that at full employment, namely, despite the entire workforce remaining employed the excess supply of l stands idle.

In fact, the institutional misapplication of ordinary rewards to ef under state capitalism, that is, the substantial contract menu $(y_{ef},\ w)$, gives rise to a negative change in s which the accidental contract menu $(y_{ef},\ w)$ of market capitalism does not, for state capitalism's labor market jurisprudence eliminates the threat of

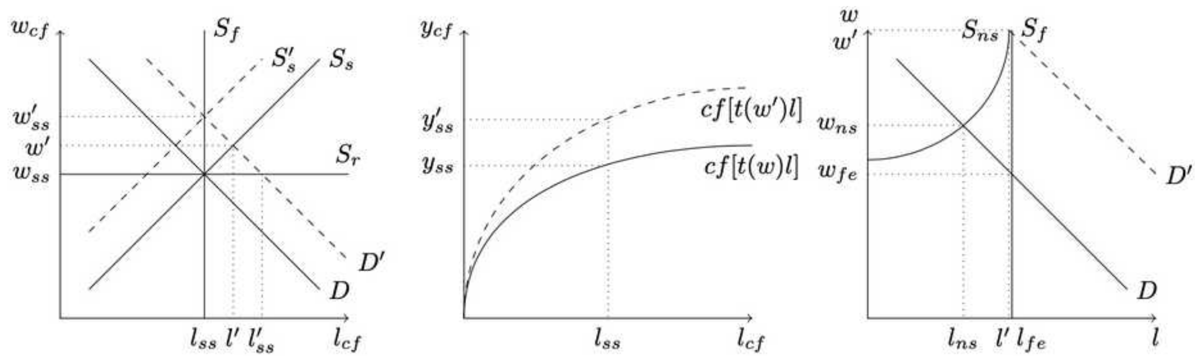
13 Workers could themselves refuse the lower w even if employers sufficiently decreased w, thereby giving rise to unemployment all the same: the two events are independent.

dismissal and unemployment[14], while market capitalism's does not (*i.e.* the contract menu $(y_{ef},\ s)$ substantially remains). The negative change in s which state capitalism triggers thus negatively affects steady state y_{ef}, exacerbating the intensive scarcity problem which so-called scientific socialism promised to resolve by means of centralized production: $-ef_t t_s < 0$. Market capitalism's dynamism is not therefore accelerated with regard to subsistence technology by state capitalism, but sabotaged.

If a positive demand shock affected y_{cf} under market capitalism then in view of the confusion of cf with ef[15] employers' unwillingness to sufficiently increase w, whereby a rise in w does not affect y_{ef}, would similarly give rise to a supply shortage of l, remedied through an expansion of the workforce by means of immigration[16], until they accommodate the higher w and return to steady state y_{cf} at pre-immigration l supply[17]; in fact, steady state y_{cf} would actually increase, on account of employers' belated rise in w. Under state capitalism said supply shortage of l would be immediately accommodated at institutional w, via international immigration, causing full employment y_{cf} to permanently increase.

The institutional misapplication of pain incentives to cf by market capitalism's labor market jurisprudence, by which the threat of dismissal and unemployment is transversal across production kinds[18], offering the substantial contract menu $(y_{cf},\ s)$, contrarily excluded by state capitalism's substantial contract menu $(y_{cf},\ w)$, does not give rise to a negative change in w, however, affecting not steady state y_{cf}: $-cf_t t_w < 0$ is not verified. An orthodox interpretation of the physiocratic school (*i.e.* state abstentionism even in worker rights) thus features institutional anxiety, but does not thereby derail luxury technology, for cajolement is present as well (optimally and not).

Figure 2. Positive demand shock and institutional effects on y



Note. The first graph depicts the contract menu (y_{cf}, s) and (y_{cf}, w) under market and state capitalism, respectively, being (i) accidental and substantial and (ii) substantial. Under market capitalism a positive demand shock to y_{cf} increases l demand and w therewith (*i.e.* D' and w'), but employers confuse y_{cf} with y_{ef} and thereby refuse to sufficiently increase w in order for l_{ss} to be preserved, fearing steady state y_{cf} may remain unvaried as if it were y_{ef}. The l supply shortage (*i.e.* l') is thus cleared through immigration, which eventually recedes, the workforce returning to l_{ss} through a fall in l supply (*i.e.* S'_s), at higher w'_{ss}, which in turn causes steady state y_{cf} to rise, as the second graph depicts (*i.e.* y'_{ss} due to w', from steady state w). Such is in contrast with the Shapiro and Stiglitz (1984) no shirking condition, depicted in the third graph: however much may l demand rise offered w will ever exclude full employment (*i.e.* l' at w', on D'). Under state capitalism the positive demand shock increases l demand and since w are institutionally rigid (*i.e.* price rigidity, l supply S_r) l_{ss} and steady state y_{cf} permanently increase (*i.e.* w_{ss} at l'_{ss}).

14 As capital is rented by the state in view of effortful production negotiation costs do not arise, being there nothing to intrinsically negotiate, but y_{ef} decreases. Instead of being subjected to the threat of dismissal, to wit, the instantiated cultivator is inexorably paid his salary, is left unsupervised and entrusted with the demesne altogether, but because the public landowner inspects the demesne negotiation costs are absent; the marginal product of his dismissal risk nevertheless decreases.

15 The contract menu $(y_{cf},\ s)$ is both accidental and substantial. In other words, y_{cf} is optimally induced through w, but being exchanged with y_{ef} the contract menu $(y_{cf},\ s)$ accidentally arises; indeed, the transversality across production kinds of the threat of dismissal and unemployment featured by market capitalism renders the contract menu $(y_{cf},\ s)$ substantial as well.

16 Immigration could be intra or inter-sectoral, domestically or thereby from abroad.

17 Immigration is implicitly surmised to recede, because of employers' procrastination in sufficiently increasing w to the end of accommodating risen demand.

18 As labor is hired by employers in view of careful production supervision costs do not arise, being there nothing to intrinsically supervise, and y_{cf} does not decrease. Other than being paid his commission, to wit, the instantiated tailor is entrusted with the fabric to be sewn, but because he infuses his skill into the fabric as the sewing progresses supervision costs are absent; the tailor is not moreover imputed the risk of fabric damage, for it is not negotiated. The tailor is also subjected to the threat of dismissal, but the marginal product of his commission does not thereby decrease.

A negative and positive demand shock to y_{cf} and y_{ef}, respectively, under market capitalism absent confusion in production kinds would moreover cause employers to (i) unwillingly decrease w, by contrast necessary in order for steady state y_{cf} to return, indeed ultimately lower precisely because of the decrease of t in $-w$ thereby, and (ii) willingly increase w to an analogous end, namely, the return of steady state y_{ef}, indeed attainted to, t not being a function of w thereby. The same shocks under state capitalism would finally only give rise to a negative and positive change in steady state y_{cf} and y_{ef}, respectively, w institutionally not even varying again.

Conclusion

In a setting of labor market imperfect competition the formal differentiation of (i) pain incentives from ordinary rewards, (ii) of effortful from careful production and (iii) of diligent from slothful workers suggests that: (i) diligent workers trigger indifference between pain incentives and ordinary rewards as inducements for both effortful and careful production; (ii) slothful workers trigger preference for either pain incentives or ordinary rewards as inducements for production in accordance with production kind, being suitably effortful or careful. It furthermore suggests that the optimal menu of contracts thereby associates inducements to production kinds following the preference triggered by slothful workers: effortful production hires labor and prescribes pain incentives; careful production rents capital and prescribes ordinary rewards. The efficiency of the efficiency wage as interpreted by the sociological theory is therefore discerned to arise under (and perhaps only under) a particular production kind and so is that of slavery its dual (impinging by no means on its illicitness, of course). More broadly, the confusion of the two production kinds under market and state capitalism respectively contributes to the Phillips curve and price rigidity, in the misapplication of ordinary rewards to effortful production. State capitalism jurisprudentially eliminates the risks of dismissal and redundancy and thereby lastly causes effortful production to enter stagnation.

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