Why Is Risk Aversion Essentially Important for Endogenous Economic Growth?

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Abstract:

The familiar condition for a balanced growth path indicates that a household's attitude toward risk plays a significantly important role for endogenous economic growth, but the mechanism behind this importance has not been sufficiently examined.
In this paper, I show that in the process of endogenous growth, the decreasing rate of marginal utility is kept constant and the household's quickness of response to new technologies determines the growth rate. Quickness of response to new technology and degree of risk aversion are quite similar. Given a constant decreasing rate of marginal utility, if on average households in a country are more cautious and respond less quickly to new technologies, firms in that country will invest less in new technologies. As a result, the endogenous economic growth rate of the country will be lower than that of others. If people respond more quickly, the growth rate will be higher.

Keywords: decreasing rate of marginal utility; endogenous economic growth; risk aversion.

JEL Classification: D81; O40.

Introduction

The importance of a household's attitude toward risk in endogenous economic growth can be easily understood. Given a Harrod-neutral production function such that $y_t = A_t^{\alpha} k_t^{1-\alpha}$ and a power utility function $u(c_t)$, the familiar optimal growth path is:

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left[(1-\alpha)\left(\frac{A_t}{k_t}\right)^{\alpha} - \theta\right] \tag{1}$$

where: $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $c_t = \frac{C_t}{L_t}$, Y_t (≥ 0) is output; K_t (≥ 0) is capital input; L_t (≥ 0) is labor input; A_t (≥ 0) is technology; C_t (≥ 0) is consumption in period t.

In addition, θ is the rate of time preference (RTP), α ($0 < \alpha < 1$) is a constant, ε is the degree of relative risk aversion (DRA), and $\varepsilon = -\frac{c_t \frac{d^2u}{dc_t^2}}{\frac{du}{dc_t}}$. Equation (1) clearly indicates that if $\frac{A_t}{k_t}$ is kept constant, the growth rate $\frac{\dot{y}_t}{y_t}$ is constant, and in addition, if:

$$(1-\alpha)\left(\frac{A_t}{k_t}\right)^{\alpha} - \theta > 0 \,, \tag{2}$$

the economy grows on a balanced growth path at a positive constant rate. An important point in equation (1) is that the growth rate $\frac{\dot{y}_t}{y_t}$ crucially depends on ε (*i.e.*, the household's attitude toward risk).

There are various types of endogenous growth models, and most of them are common in that they are constructed on the basis of mechanisms that make $\frac{A_t}{k_t}$ constant and inequality (2) hold, although they rely on different mechanisms (*e.g.,* Romer 1986, 1987, 1990, Lucas 1988, Grossman and Helpman 1991, Aghion and Howitt 1992, 1998, Jones 1995, 1999, Kortum 1997, Segerstrom 1998, Eicher and Turnovsky 1999, Young 1998, Peretto 1998, Dinopoulos and Thompson 1998l Peretto and Smulders 2002, Harashima 2019b)[1]. However, most

[1] Harashima (2019b) is also available in English as Harashima (2013).

of these studies have focused on the mechanism that makes $\frac{A_t}{k_t}$ constant and paid little attention to the importance of DRA (ε) in endogenous growth.

Nevertheless, a few studies have focused on the effect of risk attitude on growth (*e.g.*, García-Peñalosa and Wen 2008, Zeira 2011, Burton 2015, Ghiglino and Tabasso 2016), but most of these studies focused on the risk attitudes of researchers or entrepreneurs, not households. Indeed, how researchers or entrepreneurs respond to risks will likely affect research activities, and if researchers or entrepreneurs are less risk averse, they will engage in even riskier research projects and therefore generate a larger amount of innovations. As a result, higher economic growth will be realized. In this sense, the studies focusing on the risk attitude of researchers or entrepreneurs make sense. However, ε in equation (1) is not the DRA of a researcher or entrepreneur; it is the DRA of a household. As a whole, the mechanism behind the importance of household DRA in endogenous economic growth has been almost neglected in economic studies. The purpose of this paper is to examine this neglected mechanism and uncover the reason why household DRA plays an essential role in endogenous economic growth.

This paper examines the nature of a household's attitude toward risk in a model that is not constructed on the basis of the conventionally assumed procedure whereby households reach steady state by generating rational expectations using RTP (the "RTP-based procedure"). Rather, the model is constructed on the basis of an alternative procedure to reach steady state that I call the "MDC-based procedure" and present in Harashima (2019a)[2]. We use this model because:

- the motivation behind household actions with regard to risks are more clearly understood;
- the rational expectations hypothesis has been criticized for imposing substantial demands on economic agents.

The rational expectations hypothesis has been predominant in economics since it was popularized by Lucas (1972) and Sargent *et al.* (1973), whose papers were both based on that of Muth (1961). However, to generate rational expectations, households are assumed to do something equivalent to computing complex large-scale non-linear dynamic macro-econometric models. Can a household routinely do such a thing in its daily life? Evans and Honkapohja (2001) argued that this problem can be solved by introducing a learning mechanism (see also, *e.g.,* Marcet and Sargent 1989, Ellison and Pearlman 2011), but this solution is not necessarily regarded as being sufficiently successful because arbitrary learning rules have to be assumed.

The MDC-based procedure is very simple. A household only has to subjectively estimate its self-assessed value of the combination of its earned (labor) income and wealth (capital) (the capital-wage ratio; CWR) and then to adjust its consumption to the point at which it feels most comfortable (the maximum degree of comfortability; MDC). A household is not required to do anything equivalent to computing a complex large-scale macro-econometric model to generate rational expectations, and furthermore, it is not even required to be aware of any sort of economic model. The economy naturally reaches a steady state that can be interpreted as the same steady state reached by the RTP-based procedure.

In this paper, I show that the decreasing rate of marginal utility is kept constant by factors in the process of production, and the household's quickness of response to new technologies determines the growth rate. This quickness of response and risk aversion are two sides of the same coin, and the quickness of response is heterogeneous across households because the degree of risk aversion is heterogeneous. Given a constant decreasing rate of marginal utility, if the average response of households to new technologies is less quick in one country than another, firms of that country invest less in new technologies and as a result, the endogenous economic growth rate of the country is lower. If the response is quicker, the growth rate is higher.

1. MDC-based procedure

The MDC-based procedure and its nature are explained briefly following Harashima (2019a).

1.1 "Comfortability" of the capital-wage ratio

Let k_t and w_t be per capita capital and wage (labor income), respectively, in period t. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\breve{w}_t}{\breve{k}_t}$, where $\breve{k}_t$ and $\breve{w}_t$ are the k_t and w_t of the household, respectively. Let Γ be the household's subjective valuation of $\frac{\breve{w}_t}{\breve{k}_t}$ and Γ_i be the value of $\frac{\breve{w}_t}{\breve{k}_t}$ of household i (i = 1, 2, 3, … , M). The household should next assess whether it feels comfortable with its current Γ, that is, its combination of income and capital. "Comfortable" in this context means at ease, not anxious, and other

[2] Harashima (2019a) is also available in English as Harashima (2018).

similar related feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable CWR value, because the household will feel less comfortable if its CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC), and let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ therefore indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is the Γ_i of household *i* at $\tilde{s}_i$.

1.2 Homogeneous population

Suppose first that all households are identical (*i.e.*, a homogeneous population).

Rules

Household *i* should act according to the following rules:

Rule 1-1: If household *i* feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any *i*.

Rule 1-2: If household *i* feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any *i*.

Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period *t*, and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (*i.e.*, the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Also, let $\tilde{S}_{RTP}$ be the steady state under an RTP-based procedure, that is, one derived in a Ramsey-type growth model in which households behave by discounting utilities by θ and generating rational expectations, where $\theta(> 0)$ is the household's rate of time preference (RTP), and let $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ under the RTP-based procedure in an economy where θ is identical for all households, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.

Proof: See Harashima (2019a).

Proposition 1 indicates that we can interpret that $\tilde{S}_{MDC}$ is equivalent to $\tilde{S}_{RTP}$. This means that both procedures can function equivalently and that CWR at MDC is substitutable for RTP as a guide for household behavior.

1.3 Heterogeneous Population

In actuality, households are not identical - they are heterogeneous - and if heterogeneous households behave unilaterally, there is no guarantee that a steady state other than corner solutions exists (Becker 1980, Harashima 2012, 2017)[3]. However, Harashima (2012, 2017) showed that a sustainable heterogeneity (SH) at which all optimality conditions of all heterogeneous households are simultaneously satisfied exists under the RTP-based procedure. In addition, Harashima (2019a) showed that SH also exists under the MDC-based procedure, although Rules 1-1 and 1-2 have to be revised and a rule for the government must be added in a heterogeneous population.

Suppose that households are identical except for their CWRs at MDC (*i.e.*, their values of $\Gamma(\tilde{s})$). Let $\tilde{S}_{MDC,SH}$ be the steady state at which MDC is achieved and kept constant by any household (*i.e.*, SH in a heterogeneous population under the MDC-based procedure), and let $\Gamma(\tilde{S}_{MDC,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC,SH}$. In addition, let Γ_R be a household's numerically adjusted value of Γ for SH based on the information it has about its estimated values of $\Gamma(\tilde{S}_{MDC,SH})$. Specifically, let $\Gamma_{R,i}$ be Γ_R of household *i*. Let also *T* be the net transfer that a household receives from the government with regard to SH. Specifically, let T_i be the net transfer that household *i* receives ($i = 1, 2, 3, \ldots, M$).

[3] Harashima (2017) is also available in English as Harashima (2010).

Revised and Additional Rules

Household *i* should act according to the following rules in a heterogeneous population:

Rule 2-1: If household *i* feels that the current $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption as before for any *i*.

Rule 2-2: If household *i* feels that the current $\Gamma_{R,i}$ is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption or revises its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ so that it perceives that $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$ for any *i*.

At the same time, a government should act according to the following rule:

Rule 3: The government adjusts T_i for some *i* if necessary so as to make the number of votes cast in elections in response to increases in the level of economic inequality equivalent to that in response to decreases.

Steady state

Even if households and the government behave according to Rules 2-1, 2-2, and 3, there is no guarantee that the economy can reach $\tilde{S}_{MDC,SH}$. However, thanks to the government's intervention, SH can be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be the state at which $\tilde{S}_{MDC,SH}$ is approximately achieved (see Harashima 2019a), and $\Gamma(\tilde{S}_{MDC,SH,ap})$ be $\Gamma(S_t)$ at $\tilde{S}_{MDC,SH,ap}$ on average. Here, let $\tilde{S}_{RTP,SH}$ be the steady state that satisfies SH under the RTP-based procedure when households are identical except for their RTPs. In addition, let $\Gamma(\tilde{S}_{RTP,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP,SH}$.

Proposition 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, if the government behaves according to Rule 3, and if the value of θ_i that is calculated back from the values of variables at $\tilde{S}_{MDC,SH,ap}$ is used as the value of θ_i for any *i* under the RTP-based procedure in which households are identical except for their RTPs, then $\Gamma(\tilde{S}_{MDC,SH,ap}) = \Gamma(\tilde{S}_{RTP,SH})$.

Proof: See Harashima (2019a).

Proposition 2 indicates that we can interpret that $\tilde{S}_{MDC,SH,ap}$ is equivalent to $\tilde{S}_{RTP,SH}$. No matter what values of T, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are severally estimated by households, any $\tilde{S}_{MDC,SH,ap}$ can be interpreted as the objectively correct and true steady state. In addition, a government need not necessarily provide the objectively correct T_i for $\tilde{S}_{MDC,SH,ap}$, even though the $\tilde{S}_{MDC,SH,ap}$ is interpreted as objectively correct and true.

2. Technological Progress under the MDC-Based Procedure

2.1 Response to Technological Progress under the MDC-Based Procedure

2.1.1 Response to Technological Progress

Harashima (2019a) showed how a household responds to technological progress under the MDC-based procedure as follows:

- Channel (a): If a new version of a product with higher performance at almost the same price as the old version is introduced, a household will buy the new version instead of the old version while its MDC is unchanged.
- Channel (b): If a household's income unexpectedly and permanently increases, the household begins to feel that its current Γ is unexpectedly higher than $\Gamma(\tilde{s})$. However, because of the increase in income, its capital unexpectedly gradually increases, and the household will leave this accumulation of capital as it is until its Γ is returned to its $\Gamma(\tilde{s})$.

Technological progress thereby causes the economy to grow through the household's responses via Channels (a) and (b).

2.1.2. Effect on Investments in Technologies

If technologies are only given exogenously, the quickness of households' response to new technologies through Channels (a) and (b) will not ultimately affect economic growth. If technologies are endogenously generated, however, the quickness of households' response will have an important effect on growth because firms have to make decisions on investments in new technologies fully considering how households will respond to new technologies.

If households respond less quickly, fewer new products with new technologies will be purchased by households in a unit period. Firms therefore will be more cautious about investments in new technologies because they may not obtain sufficient returns from the investments or, even worse, suffer losses. As a result, if households respond less quickly, the speed of technological progress and thereby the growth rate of the economy will be lower.

2.2 The Utility Function

Under the MDC-based procedure, households feel the utilities from consumption in a similar manner as they do under the RTP-based procedure. Under the MDC-based procedure, μ is a function of the level of current or future consumption estimated by the household (c_l). It is important to note that c_l is a simply estimated value, and the expected μ is not discounted by RTP.

Suppose a usual power utility function such that:

$$\mu = \frac{c_l^{1-\delta}}{1-\delta} \text{ if } \delta \neq 1 \mu = \ln c_l \text{ if } \delta = 1$$

where: δ (≥ 0) is a parameter. Therefore,

$$\delta = -\frac{c_l \frac{d^2\mu}{dc_l^2}}{\frac{d\mu}{dc_l}} (> 0) \ . \tag{3}$$

Note that δ can be interpreted to be equivalent to DRA under the RTP-based procedure.

2.3 The Decreasing Rate of Marginal Utility

2.3.1 Constancy

By equation (3),

$$\frac{\dot{c}_l}{c_l} = -\delta^{-1} \frac{\frac{d\left(\frac{d\mu}{dc_l}\right)}{dt}}{\frac{d\mu}{dc_l}} \ . \tag{4}$$

Let the marginal utility be υ; thereby, for utility μ, $\upsilon = \frac{d\mu}{dc_l}$,

and the decreasing rate of marginal utility is: $-\frac{\frac{d\left(\frac{d\mu}{dc_l}\right)}{dt}}{\frac{d\mu}{dc_l}} = -\frac{\dot{\upsilon}}{\upsilon} (> 0)$.

Hence, by equation (4),

$$\frac{\dot{c}_l}{c_l} = -\delta^{-1} \frac{\dot{\upsilon}}{\upsilon} \tag{5}$$

On a balanced growth path,

$$\frac{\dot{y}}{y} = \frac{\dot{c}_l}{c_l} = \text{constant.} \tag{6}$$

Therefore, by equation (5), for any given value of δ, $-\frac{\dot{\upsilon}}{\upsilon}$ = constant on a balanced growth path.

2.3.2. Constant Deceasing Rate of Marginal Utility

Equation (5) indicates that, for a given value of δ, if the growth rate of the economy $\frac{\dot{y}}{y}$ is constant (*i.e.*, the economy is on a balanced growth path), the growth rate is uniquely determined by the value of $-\frac{\dot{\upsilon}}{\upsilon}$, and as the value of $-\frac{\dot{\upsilon}}{\upsilon}$ increases, the constant growth rate $\frac{\dot{y}}{y}$ increases. Conversely, a higher economic growth rate is accompanied by a higher decreasing rate of marginal utility.

It seems highly likely that most households prefer higher economic growth rates, and furthermore, they want the growth rate to be as high as possible. However, in actuality, the long run growth rate of an economy has an upper bound. Equations (5) and (6) imply that this upper bound originates in the constant or fixed deceasing rate of marginal utility $-\frac{\dot{\upsilon}}{\upsilon}$. Let Υ be this constant $-\frac{\dot{\upsilon}}{\upsilon}$.

Why is Υ constant? There are two possibilities: (a) it originates in the household's state of mind and is a kind of household preference, and (b) it is bound by some factors in the production process. Ifhouseholds intrinsically dislike decreases in marginal utility, possibility (a) may be the reason why Υ is constant. A decrease in marginal utility means a household feels somewhat saturated with consumption. Going past the point of saturation or satiation will most likely negatively influence a household's state of mind. If households really decreases in marginal utility, they face the dilemma of choosing between a higher growth rate and a higher decreasing rate of marginal utility because the former inevitably accompanies the latter if households prefer higher economic growth rates. As a result, households will pursue higher economic growth rates only as long as they do not perceive that the decreasing rate of marginal utility is too high; that is, they do not feel that it exceeds the upper bound, which is the constant Υ.

Historically, however, persistently high economic growth rates (*e.g.*, 10% annually over decades) and persistently low economic growth rates (*e.g.,* less than 1% annually over decades) have been observed in some economies and in some periods. The high rates have usually been observed when a large amount of new technologies have been continuously introduced from one country or countries to another (*e.g.*, during a catch-up period in developing economies). This means that, if possible, households prefer or allow a high growth rate as possible, and conversely, thatΥ is not constrained by a household's state of mind or preferences. Hence, it seems likely that possibility (b) is the true reason for the constant Υ. This issue is discussed in more detail in Section 3.4.

3. Substitutability

3.1. Endogenous Growth under the RTP-based Procedure

As discussed in Section 2.1.2, heterogeneity in households' quickness of response to technological progress only matters when technology is considered endogenously. Before examining the nature of endogenous economic growth under the MDC-based procedure, for comparison, I first examine it under the RTP-based procedure on the basis of the endogenous growth model presented by Harashima (2019b), which is a natural extension of a Ramsey-type growth model.

Outputs (Y_t) are the sum of consumption (C_t), the increase in capital (K_t), and the increase in technology (A_t) in period *t* such that:

$$Y_t = C_t + \dot{K}_t + v\dot{A}_t \ ,$$

where: $v(>0)$ is a constant, and a unit of K_t and v^{-1} of a unit of A_t are equivalent; that is, they are produced using the same quantities of inputs (capital, labor, and technology). The productivity of researchers to produce innovations is represented by the term v^{-1}. Thus,

$$\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t \ ,$$

where: n_t is the population growth rate. It is assumed for simplicity that $n_t = 0$, and thereby L_t is constant such that $L_t = L$ for any *t*. The production function is $y_t = A_t^{\alpha} k_t^{1-\alpha}$. For any period,

$$m = \frac{M_t}{L_t} \ ,$$

where: M_t is the number of firms (all of which are assumed to be identical) and m (> 0) is a constant. In addition, through the arbitrage between investments in k_t and A_t in markets,

$$\frac{\partial y_t}{\partial k_t} = \frac{\varpi}{mv}\frac{\partial y_t}{\partial A_t}$$

is always kept, where $\varpi(> 1)$is a constant and indicates the effect of patent protection. As a result,

$$A_t = \frac{\varpi\alpha}{mv(1-\alpha)} k_t$$

always holds, and therefore,

$$\dot{A}_t = \frac{\varpi\alpha}{mv(1-\alpha)} \dot{k}_t \ , \qquad y_t = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{-\alpha} k_t \ ,$$

and

$$\dot{k}_t = \frac{mL(1-\alpha)}{mL(1-\alpha) + \varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{-\alpha} k_t - c_t\right] \ .$$

On the other hand, the utility function of household $u(c_t)$ is:

$u = \frac{c_t^{1-\varepsilon}}{1-\varepsilon}$ if $\varepsilon \neq 1$, $\quad u = \ln c_t$ if $\varepsilon = 1$

where: ε is a positive parameter indicating DRA and $\varepsilon = -\frac{c_t \frac{d^2u}{dc_t^2}}{\frac{du}{dc_t}}$.

In addition, as with equation (5),

$\frac{\dot{c}_t}{c_t} = -\varepsilon^{-1}\frac{\dot{v}_t}{v_t}$ for marginal utility $v_t = \frac{du(c_t)}{dc_t}$.

Let Hamiltonian *H* be:

$$H = u(c_t)\exp(-\theta t) + \lambda_t \frac{mL(1-\alpha)}{mL(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_t - c_t\right], \tag{7}$$

where: θ is the RTP of household, and λ_t is a constant variable. Suppose that *L* is sufficiently large and therefore approximately

$$\frac{mL(1-\alpha)}{mL(1-\alpha)+\varpi\alpha} = 1\,. \tag{8}$$

By equations (7) and (8), the optimality conditions of household are

$$\frac{\partial u(c_t)}{\partial c_t}\exp(-\theta t) = \lambda_t \tag{9}$$

$$\dot{\lambda}_t = -\frac{\partial H}{\partial k_t} \tag{10}$$

$$\dot{k}_t = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_t - c_t\,, \qquad \lim_{t\to\infty}\lambda_t k_t = 0.$$

By equation (10),

$$\dot{\lambda}_t = -\lambda_t\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} \tag{11}$$

Hence, by equations (9) and (11), the growth rate of consumption is

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right] \tag{12}$$

This path is the balanced growth path in the model under the RTP-based procedure, and I call this model the "RTP model." By equation (11),

$$\frac{\dot{\lambda}_t}{\lambda_t} = -\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}\,. \tag{13}$$

By equations (9) and (13), the marginal utility $v_t = \frac{du(c_t)}{dc_t}$ decreases at a constant rate:

$-\frac{\dot{v}_t}{v_t} = -\frac{\frac{d\left[\frac{du(c_t)}{dc_t}\right]}{dt}}{\frac{du(c_t)}{dc_t}} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta$; that is, the decreasing rate of marginal utility is the same as the marginal productivity of capital minus RTP. This is the condition for a balanced growth path with regard to the marginal utility under the RTP-based procedure.

Let G_t be the growth path of the economy in period *t* and $\Psi(G_t)$ be the average growth rate of the economy on G_t. In addition, let $\tilde{G}_{RTP}$ be the balanced growth path in the RTP model, and $\Psi(\tilde{G}_{RTP})$ be $\Psi(G_t)$ for $G_t = \tilde{G}_{RTP}$.

3.2. Endogenous Growth under the MDC-based Procedure

Next, I examine the nature of endogenous growth under the MDC-based procedure. Households keep,

$\Gamma(S_t) = \frac{w_t}{k_t} = \alpha\frac{y_t}{k_t} = \alpha\left(\frac{A_t}{k_t}\right)^{\alpha}$ = constant under the MDC-based procedure by behaving according to Rule 1-1 and 1-2 (or 2-1 and 2-1); that is, $\frac{A_t}{k_t}$ is kept constant (as assumed above, the production function is $y_t = A^{\alpha} k_t^{1-\alpha}$). In this sense, a balanced growth path can be naturally achieved under the MDC-based procedure. Furthermore, because households prefer higher rates of economic growth, $-\frac{\dot{v}}{v}$ increases up to the point $-\frac{\dot{v}}{v} = \Upsilon$, but it stops increasing at this level by reason of possibility (a) or (b) in Section 2.3.2, and by equation (5), the growth rate $\frac{\dot{c}_l}{c_l} = \frac{\dot{y}}{y}$ becomes constant. As equation (6) indicates, this growth path is clearly a balanced growth path. This means that a balanced growth path is naturally achieved through the behavior of households with the MDC-based procedure. I call this the "MDC model."

Here, suppose for simplicity that all households are identical. Let $\tilde{G}_{MDC}$ be a balanced growth path and $\Psi(\tilde{G}_{MDC})$ be $\Psi(G_t)$ when $G_t = \tilde{G}_{MDC}$. The production function is the same as that in the previous sections (*i.e.*, $y_t = A^{\alpha} k_t^{1-\alpha}$), and $A_t = \frac{\varpi \alpha}{mv(1-\alpha)} k_t$ is kept through arbitrage in markets. Households and firms prefer higher growth rates, other things being equal.

Lemma 1: If all households are identical and behave according to Rules 1-1 and 1-2, then $\tilde{G}_{MDC}$ exists.

Proof: Because all households are identical and behave according to Rules 1-1 and 1-2, then by Harashima (2019a), $\tilde{S}_{MDC}$ exists, and even if A_t changes, $\tilde{S}_{MDC}$ is soon restored (achieved again) by the same mechanism that makes $\tilde{S}_{MDC}$ exist.

Because households and firms prefer higher growth rates, firms invest in technologies as much as possible up to the level that corresponds to the constant Υ. Because all households are identical, their values of Υ and δ are also identical. Because both Υ and δ take only one finite value in any period, respectively, then $\frac{\dot{c}_l}{c_l}$ takes a finite value in any period, and therefore the economy grows on average at a finite rate.

Because $\tilde{S}_{MDC}$ is restored even if A_t changes, $\tilde{S}_{MDC}$ is basically held on the path along which the economy grows at a finite rate on average. Hence, this path is a $\tilde{G}_{MDC}$ and therefore $\tilde{G}_{MDC}$ exists. Q.E.D.

By equation (5), on $\tilde{G}_{MDC}$,

$$c_g = \delta^{-1} \Upsilon \tag{14}$$

where: c_g is the average $\frac{\dot{c}_l}{c_l}$ on $\tilde{G}_{MDC}$.

3.3. Substitutability Between the Two Procedures

In this section, I examine whether $\tilde{G}_{MDC}$ (*i.e.*, a balanced growth path in the MDC model) can be interpreted to be equivalent to $\tilde{G}_{RTP}$ (*i.e.*, the balanced growth path in the RTP model).

Proposition 1: Assign Υ the value that satisfies

$$\Upsilon = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{-\alpha} - \theta, \tag{15}$$

where: the values of ϖ, α, m, v, and θ are all the same as those in the RTP model. If all households are identical and behave according to Rules 1-1 and 1-2, and if the value of δ that is calculated by equation (14) based on the value of c_g on $\tilde{G}_{MDC}$ and the assigned value of Υ is used as the value of ε in the RTP model, then $\Psi(\tilde{G}_{MDC}) = \Psi(\tilde{G}_{RTP})$.

Proof: By Lemma 1, a $\tilde{G}_{MDC}$ exists. In addition, equation (12) holds for $\tilde{G}_{RTP}$. Because equation (15) holds, if the value of ε is set equal to the value of δ that is calculated by equation (14) based on the value of c_g on $\tilde{G}_{MDC}$ and the assigned value of Υ, then by equation (12),

$$\frac{\dot{c}_t}{c_t} = \delta^{-1}\Upsilon = c_g \tag{16}$$

for $\tilde{G}_{RTP}$. By equations (14) and (16), $\Psi(\tilde{G}_{MDC}) = \Psi(\tilde{G}_{RTP})$, Q.E.D.

Proposition 1 indicates that we can interpret that $\tilde{G}_{MDC}$ is equivalent to $\tilde{G}_{RTP}$. The RTP- and MDC-based procedures can function equivalently and are substitutable to reach steady state and for endogenous economic growth. It is important to note that we cannot know whether the achieved $\tilde{G}_{MDC}$ is the objectively "true" and "correct" balanced growth path. We know only that $\tilde{G}_{MDC}$ is a balanced growth path on which all households feel most comfortable on average, and we can interpret that it is equivalent to $\tilde{G}_{RTP}$.

Proposition 1 also indicates that a household can respond well to technological progress without calculating the expected discounted utility based on the "true" and "correct" value of ε. With respect to responding to technological progress, therefore, the MDC-based procedure is unquestionably far easier to use than the RTP-based procedure, and is therefore much more likely to actually be used. Harashima (2019a) also showed that the MDC-based procedure is most likely to actually be used to reach steady state for the same reason. It is highly likely therefore that households behave only feeling Γ and Υ without generating the expected discounted utility based on the values of θ and δ.

3.4. The Origin of Constant Υ

As indicated in Section 2.3.2, there are two possibilities of the origin of constant Υ. However, Proposition 1 strongly implies that the true origin is possibility (b) because equation (15) indicates that the value of Υ is exogenously determined by the values of parameters α, m, v, and ϖ on the supply side and θ. Because the values of α, m, v, and ϖ cannot be changed by households at will, if the value Υ is determined by a household's mindset and represents a household's preference, it is almost impossible for equation (15) to hold. Proposition 1 and equation (15) therefore mean that households adjust the value of Υ so as to be consistent with the values of α, m, v, and ϖ, which are determined independently, technically, and exogenously on the supply side. That is, it is highly likely that the true origin of constant Υ lies in possibility (b), as was also concluded in Section 2.3.2.

However, what factors in the process of production constrain the production of innovations? One possibility is that they are bound by the limits on the productivity of researchers to produce innovations represented by the term v^{-1} in equation (12). As the productivity v^{-1} increases, the economic growth rate increases. As with δ, therefore, v^{-1} will be an important determinant of endogenous economic growth.

Another possibility is that households adjust the value of θ so as to be consistent with those of α, m, v, ϖ, and Υ for equation (15) to hold. However, because both θ and Υ are determined by households, the reason why θ has to be subordinated to Υ is difficult to explain.

3.5. The Role of δ in Economic Growth

Section 2.3 indicates that given a common constant Υ, the economic growth rate and equivalently the speed of technological progress depend on the value of δ. The speed of technological progress is determined by the amount of investments in new technologies in a unit period, and as indicated in Section 2.1.2, this amount is affected by household's quickness of response to new technologies. This means that the value of δ is equivalent to the degree of a household's quickness of response to new technologies. In other words, the value of δ indicates the degree of a household's quickness of response about new technologies.

A higher value of δ indicates that households are more cautious about new technologies, and therefore, the responses through Channels (a) and (b) are less quick. If households are more cautious about new technologies, firms can obtain lower returns from investments in new technologies because products will not sell as expected, which will result in smaller amounts of investments in new technologies in a unit period and thereby lower the rate of economic growth. That is, as the value of δ increases, the endogenous economic growth rate decreases and vice versa.

In the RTP model, ε indicates DRA (*i.e.,* an indicator of a household's attitude toward risk). Because δ can be interpreted to be equivalent to ε, a household's cautiousness about new technologies can be interpreted to be equivalent to a household's attitude toward risk. The equivalence indicates that a household's DRA is essentially important for the endogenous economic growth model.

4. Heterogeneous δ and Substitutable Heterogeneity

4.1. Heterogeneity in δ

In Sections 2 and 3, all households are assumed to be identical for simplicity, but households are actually heterogeneous, and if the constant Υ is common to all households, different values of δ indicate some heterogeneity among households. In this section, I examine the case that households are heterogeneous in δ.

Suppose that there are only two economies (*economy 1* and *economy 2*), where an economy means a group of households in a country. Both economies consist of the same number of households. Households in the two economies are identical except for the values of δ, and $\delta_1 < \delta_2$ where δ_i is the δ of a household in *economy i* (= 1 or 2) and households within each economy are identical. Because households are heterogeneous only in δ, the constant Υ is identical for all households such that:

$$\Upsilon_i = \bar{\Upsilon} > 0$$

for any *i* where Υ_i is the Υ of a household in *economy i*.

The two economies are fully open to each other except for the labor force, and capital moves completely elastically so that the marginal product of capital is kept equal through arbitrage in markets. Hence, the amount of capital operating in each economy is always identical (*i.e.*, $k_1 = k_2$) and thereby household wages in both economies are also always identical such that $w_1 = w_2$. The amounts of capital owned by a household in the two economies can be different, but they are assumed to be the same in the initial period *t*.

4.2. The Case without Government Intervention

Under the MDC-based procedure, a household increases capital supposing that production, capital, and technology will increase at the same rate as its consumption because the household behaves as if its CWR is kept equal to the level at MDC. Therefore, by equation (5), in the initial period *t*, a household in *economy* 1 increases the amount of capital it owns at the rate:

$$\frac{\dot{c}_{l,1}}{c_{l,1}} = \delta_1^{-1}\bar{\Upsilon} , \tag{17}$$

and at the same time, a household in *economy 2* increases the amount of capital it owns at the rate:

$$\frac{\dot{c}_{l,2}}{c_{l,2}} = \delta_2^{-1}\bar{\Upsilon} , \tag{18}$$

where: $c_{l,1}$ and $c_{l,2}$ are the c_l of households in *economies 1 and 2,* respectively. On the other hand, in the same period *t*, wages (w_1 and w_2) commonly increase at the average rate:

$$\frac{\frac{\dot{c}_{l,1}}{c_{l,1}} + \frac{\dot{c}_{l,2}}{c_{l,2}}}{2} = \left(\frac{\delta_1 + \delta_2}{2\delta_1\delta_2}\right)\bar{\Upsilon} \tag{19}$$

in both economies by equations (17) and (18) because increases in capital and technologies operating in each economy are always kept identical through arbitrage; therefore, $w_1 = w_2$ is always maintained. Because,

$$\delta_1^{-1}\bar{\Upsilon} - \frac{\delta_1 + \delta_2}{2\delta_1\delta_2}\bar{\Upsilon} = -\frac{\delta_1 - \delta_2}{2\delta_1\delta_2}\bar{\Upsilon} > 0$$

and

$$\delta_2^{-1}\bar{\Upsilon} - \frac{\delta_1 + \delta_2}{2\delta_1\delta_2}\bar{\Upsilon} = \frac{\delta_1 - \delta_2}{2\delta_1\delta_2}\bar{\Upsilon} < 0 ,$$

equations (17), (18), and (19) indicate that the rate of increase in capital owned by households in *economy* 1 is higher than the rate of increase rate for wages, but the opposite is true for households in *economy* 2.

As a result, at the beginning of period *t* + 1, the CWR of households in *economy* 1 is lower than the level at MDC, but the CWR of households in *economy* 2 is higher than the level at MDC. Note that because all households in both economies are assumed to be identical except for their values of *δ*, CWR at MDC is identical for all households.

Because CWR is not equal to CWR at MDC, households in *economy* 1 begin to gradually reduce the excess capital generated in period *t* to restore MDC. Meanwhile, households in *economy* 2 begin to gradually increase their capital accumulation to make up for the shortage of capital generated in period *t* to restore MDC.

However, before they can fully reduce the excess capital generated in period *t*, households in *economy* 1 again begin to accumulate capital at a higher rate than the wage increases in period *t* + 1 for the same reason as in period *t*. At the same time, before households in *economy 2* can fully make up for the shortage of capital generated in period *t*, they again begin to accumulate capital at a lower rate than that of wage increases in period *t* + 1 for the same reason as in the prior period. Therefore, at the beginning of period *t* + 2, the CWR of households in *economy 1* is even lower than the level at MDC, and that in *economy 2* becomes even higher.

In period *t* + 2, in addition to still gradually reducing the excess capital generated in period *t*, households in *economy 1* are simultaneously starting to gradually reduce the excess capital generated in period *t* + 1. Households in *economy* 2 behave similarly but act to increase the rate of accumulation. These responses are repeated indefinitely; therefore, the CWR of *economy* 1 continues to decrease and that of *economy* 2 continues to increase. That is, households in both economies can never restore their MDC and SH cannot be achieved. This means that if households are heterogeneous in δ, appropriate interventions of the government of the country are needed for SH to be achieved.

4.3. SH with Government Intervention

4.3.1. SH with Government Intervention in the RTP Model

Before examining the necessary government intervention for SH in the MDC model, I first briefly explain SH with government intervention on the basis of RTP models in Harashima (2012, 2017). Again suppose two economies that consist of the same number of identical households (Economies 1 and 2). Households are identical except for DRA (ε), and let ε_1 and ε_2 be the ε of households in Economies 1 and 2, respectively, and $\varepsilon_1 < \varepsilon_2$. The government of the country intervenes by transferring money from households in *economy* 1 to those in *economy* 2. The amount of transfer in period *t* is g_t, and it is assumed that g_t depends on capital such that:

$$g_t = \bar{g}_t k_{1,t} \ ,$$

where: $\bar{g}_t$ is the ratio $\frac{g_t}{k_{1,t}}$ that is exogenously set by the government in period *t*, and the value of $\bar{g}_t$ is appropriately adjusted by the government in every period so as to achieve SH.

If the government intervenes such that:

$$\lim_{t\to\infty} \bar{g}_t = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right] , \tag{20}$$

then

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \frac{2}{\varepsilon_1+\varepsilon_2}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right] \tag{21}$$

and SH is achieved, and the combined economy of *economies 1 and 2* (*economy* 1+2) proceeds on a balanced growth path where $c_{1,t}$ and $c_{2,t}$ are the c_t of households in *economies 1 and* 2, respectively. Let $\tilde{G}_{RTP,SH}$ indicate this balanced growth path, and $\Psi\left(\tilde{G}_{RTP,SH}\right)$ be the growth rate on $\tilde{G}_{RTP,SH}$.

4.3.2. SH with Government Intervention in the MDC Model

As shown in Section 1.3.1, in a heterogeneous population, the government behaves according to Rule 3; that is, the government takes measures to make the number of votes cast in elections in response to increases in the level of economic inequality equivalent to that in response to decreases. As a result, SH can be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be this approximately achieved SH when households behave under the MDC-based procedure (see Harashima 2012, 2017).

As long as the government intervenes according to Rule 3, even if A_t changes, $\tilde{S}_{MDC,SH,ap}$ is soon restored *(i.e.*, achieved again), as indicated in the proof of Lemma 1. That is, $\tilde{S}_{MDC,SH,ap}$ is basically kept on $\tilde{G}_{MDC}$ if the government intervenes according to Rule 3. Let $\tilde{G}_{MDC,SH,ap}$ be the $\tilde{G}_{MDC}$ that is achieved by the government intervention in a heterogeneous population, and $\Psi\left(\tilde{G}_{MDC,SH,ap}\right)$ be the average growth rate of the economy on $\tilde{G}_{MDC,SH,ap}$.

4.3.3. Substitutability

By equation (21), Economies 1 and 2 on $\tilde{G}_{RTP,SH}$ can be interpreted to be an integrated economy, that is, *economy 1+2* with RTP $\frac{\varepsilon_1+\varepsilon_2}{2}$, in the RTP model. Also in the MDC model, the two economies can be interpreted to be integrating on $\tilde{G}_{MDC,SH,ap}$ as *economy 1+2*. The value of δ is δ_{1+2}, and therefore equation (14) can be rewritten as:

$$c_{g,\ SH,\ ap} = \delta_{1+2}^{-1}\bar{Y}, \tag{22}$$

where: $c_{g,\ SH,\ ap}$ is the average $\frac{\dot{c}_t}{c_t}$ of *economy 1+2* on $\tilde{G}_{MDC,SH,ap}$.

Here, suppose that the government intervenes according to Rule 3 in the MDC model, and therefore SH is approximately achieved and kept. In addition, SH is also achieved and kept by appropriate government interventions in the RTP model.

Corollary 1: Assign $\bar{Y}$ the value that satisfies:

$$\bar{Y} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha} (1-\alpha)^{-\alpha} - \theta, \tag{23}$$

where: the values of ϖ, α, m, v, and θ are all the same as those in the RTP model. If all households are identical except for δ_i or ε_i and behave according to Rules 1-1 and 1-2 in the MDC model, and if the value of δ_{1+2} that is calculated by equation (22) based on the value of $c_{g,\,SH,\,ap}$ and the assigned value of $\bar{Y}$ is used as the value of $\frac{\varepsilon_1+\varepsilon_2}{2}$ in the RTP model, then $\Psi(\tilde{G}_{MDC,SH,ap}) = \Psi(\tilde{G}_{RTP,SH})$.

Proof: Because SH is approximately achieved and kept by government interventions in the MDC model, a $\tilde{G}_{MDC,SH,ap}$ exists. Replace δ and ε with δ_{1+2} and $\frac{\varepsilon_1+\varepsilon_2}{2}$ in Proposition 1, respectively. Because the value of $\frac{\varepsilon_1+\varepsilon_2}{2}$ is set equal to the value of δ_{1+2}, the value of $c_{g,\,SH,\,ap}$ is identical to the value of $\frac{\dot{c}_{1+2,t}}{c_{1+2,t}}$ on $\tilde{G}_{RTP,SH}$ by equation (22) and Proposition 1, where $c_{1+2,t}$ is the c_t of *economy* 1+2. That is,

$\frac{\dot{c}_{1+2,t}}{c_{1+2,t}} = c_{g,\,SH,\,ap}$ and therefore $\Psi(\tilde{G}_{MDC,SH,ap}) = \Psi(\tilde{G}_{RTP,SH})$, Q.E.D.

Hence, we can interpret that $\tilde{G}_{MDC,SH,ap}$ is equivalent to $\tilde{G}_{RTP,SH}$ and there by is a balanced growth path.

Nevertheless, we cannot identify the values of ε_1 and ε_2 separately only from the information on the value of $c_{g,\,SH,\,ap}$ and the assigned value of $\bar{Y}$ because the values of δ_1 and δ_2 cannot be identified from that information. There are many possible combinations of δ_1 and δ_2 for a value of δ_{1+2} and thereby those of ε_1 and ε_2 for a value of $\frac{\varepsilon_1+\varepsilon_2}{2}$. Therefore, by equation (20), there are many possible values of $\lim_{t\to\infty} \bar{g}_t$. To identify the values of δ_1 and δ_2 as well as ε_1 and ε_2, additional information is needed.

One such piece of new information is the observed value of $\bar{g}_t$. Let $\bar{g}_{MDC}$ be the average value of $\bar{g}_t$ on $\tilde{G}_{MDC,SH,ap}$.

Corollary 2: Assign $\bar{Y}$ the value that satisfies equation (23), where the values of ϖ, α, m, v, and θ are all the same as those in the RTP model, assign $\frac{\delta_1+\delta_2}{2}$ the value that satisfies

$$\delta_{1+2} = \frac{\delta_1 + \delta_2}{2}\ , \tag{24}$$

and assign $\frac{\delta_2-\delta_1}{\delta_1+\delta_2}$ the value that satisfies,

$$\bar{g}_{MDC} = \left(\frac{\delta_2 - \delta_1}{\delta_1 + \delta_2}\right)\bar{Y}\ . \tag{25}$$

If all households are identical except for δ_i or ε_i and behave according to Rules 1-1 and 1-2 in the MDC model, and if the values of δ_1 and δ_2 that are calculated by equations (22), (23), (24), and (25) based on the values of $c_{g,\,SH,\,ap}$ and $\bar{g}_{MDC}$ and the assigned values of $\bar{Y}$, $\frac{\delta_1+\delta_2}{2}$ and $\frac{\delta_2-\delta_1}{\delta_1+\delta_2}$ are used as the values of ε_1 and ε_2 in the RTP model respectively, then $\Psi(\tilde{G}_{MDC,SH,ap}) = \Psi(\tilde{G}_{RTP,SH})$.

Proof: Because SH is approximately achieved and kept by government interventions in the MDC model, a $\tilde{G}_{MDC,SH,ap}$ exists, and the value of δ_{1+2} can be identified by equations (22) and (23). With the identified value of δ_{1+2}, the values of δ_1 and δ_2 can be identified by equations (24) and (25).

Because the identified values of δ_1 and δ_2 satisfy equation (24), if δ and ε are replaced with $\delta_{1+2} = \frac{\delta_1+\delta_2}{2}$ and $\frac{\varepsilon_1+\varepsilon_2}{2}$ in Proposition 1, respectively, then by Corollary 1, $\Psi(\tilde{G}_{MDC,SH,ap}) = \Psi(\tilde{G}_{RTP,SH})$, Q.E.D.

With the additional information about $\bar{g}_{MDC}$, the values of δ_1, δ_2, ε_1, and ε_2 can be identified.

An important point is that even though the values of ε_1 and ε_2 are identified, it is still unknown whether they are the "true" and "correct" values of ε_1 and ε_2. We can only say that if we use the values indicated in

Corollary 2, we can interpret $\Psi\left(\tilde{G}_{MDC,SH,ap}\right) = \Psi\left(\tilde{G}_{RTP,SH}\right)$.

Conclusion

A household's attitude toward risk significantly influences endogenous economic growth, but there have been only a few studies that have focused on this topic. Furthermore, most of the few studies that have studied risk attitudes have focused on those of researchers or entrepreneurs, not households. However, ε in equation (1) is not the DRA of researchers or entrepreneurs; it is the DRA of households. Therefore, it is the DRA of households that is essentially important in endogenous economic growth.

In this paper, I showed that the decreasing rate of marginal utility is kept constant by factors in the process of production, possibly by the productivity of producing new technologies, and that a household's quickness of response to new technologies (δ) determines the growth rate. This quickness of response and risk aversion are two sides of the same coin, and quickness of response is heterogeneous across households just as DRA is heterogeneous. Given a constant decreasing rate of marginal utility, if responses to new technologies are on average less quick in a country, firms invest less in new technologies, and as a result, the endogenous economic growth rate of the country will be lower and vice versa. This is the mechanism behind the importance of a household's attitude toward risk in endogenous economic growth.

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